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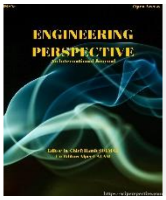

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Research Paper

Advanced Frequency of Thick FGM Spherical Shells with Fully Homogeneous Equation by Using TSDT and Nonlinear Varied Shear Coefficient

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ABSTRACT

The natural frequency values of free vibrations are generally dependent on properties of materials, boundary conditions, displacement types, vibration matter with respect to axes direction, environment temperature and shear correction in thick composited shells. Advanced natural frequency investigation of thick functionally graded material (FGM) spherical shells is presented by considering the advanced shear correction coefficient. When the more thicker shells are used, it is more necessary to consider the transverse shear effect on the vibrations. It is novel to consider the effects of nonlinear coefficient in third-order shear deformation theory (TSDT) on the advanced shear correction coefficient. The nonlinear coefficient term of TSDT is included the fully homogeneous equation according to the homogeneous matrix under free vibration. The numerical solution can be solved for the five degree polynomial equation derived from zero determinant of the fully coefficient matrix by using the numerical method, then the natural frequency can be obtained. Three effects of nonlinear coefficient term, environment temperature and power law index on the frequency of thick FGM spherical shells with advanced shear correction coefficient are studied. The numerical values of natural frequencies are calculated and investigated. These numerical values of natural frequencies are very important in the designs of structures used to prevent resonance in the vibration of mechanisms.

Keywords: nonlinear; TSDT; FGM; spherical shells; vibration; frequency

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1. Introduction

Free vibration frequency investigations with shear deformation effect and experimental studies in the spherical shells were presented. In 2021, Gan et al. [1] presented the free vibration of a shallow spherical shell by using the two-parameter foundation model of shear interaction between the spring elements. In 2021, Bagheri et al. [2] used the first order shear deformation theory (FSDT) of displacements to study the free vibrations of the conical–spherical functionally graded material (FGM) spherical shells. In 2021, Tang et al. [3] used the third-order shear deformation theory (TSDT) of displacements under the hygrothermal effects to obtain the frequency responses for the carbon fiber-reinforced polymer (CFRP) spherical shell panels. In 2022, Guo et al. [4] presented the FSDT of displacements and numerical spectral-Tchebyshev (ST) technique to study the free vibration of the laminated composite conical, cylindrical and spherical shells. In 2021, Gurve and Satankar [5] presented the results of free vibration for the stiffened cylindrical and spherical shells by using the finite element method. In 2020, Gong et al. [6] presented the free vibration analysis of the FGM spherical torus shells based on the Ritz method with thin FSDT of displacements. In 2020, Du et al. [7] presented the Flügge's thin shell theory and energy method to obtain the vibration solutions for the truncated spherical shells. In 2019, Ghavanloo et al. [8] presented the numerical and experimental results by using the nonlocal spherical shell model for the free vibration of spherical fullerene molecules. In 2017, Okhovat and Boström [9] presented the eigen-frequency results of vibration for an isotropic spherical shell by using the power series method.

Free vibration frequency computational experiences in the composited FGM shells were presented. In 2020, Hong [10] used the thick displacement approach of TSDT to present the results of free vibration frequency for the thick FGM spherical shells with fully homogeneous equation. It is novel further to study the natural frequency in the TSDT of thick FGM spherical shells under free vibration with fully homogeneous equation and more considering the advanced shear correction coefficient. Three effects of nonlinear coefficient term, environment temperature and power law index on the natural frequency of thick FGM spherical shells with advanced shear correction coefficient are investigated for the angle between the direction of z axis and the direction of radius in the spherical shells.

2. Formulation

Two-material thick FGM spherical shells for $0° \leqq \emptyset \leqq 90°$ in thermal environment is shown in Figure 1 with the axial length L in *x* direction, inner layer thickness h_1 of FGM material 1 and outer layer thickness h_2 of FGM material 2. A point in the coordinates $(r, \theta, \emptyset)$ correspond to (x, y, z), in which *r* is the radius, θ denotes the angle of circumferential direction, $\emptyset$ denotes the angle between direction of z axis and direction of *r* axis. The power-law material property functions of FGM spherical shells with power law index R_n and in functions of environment temperature T are used [11-12].

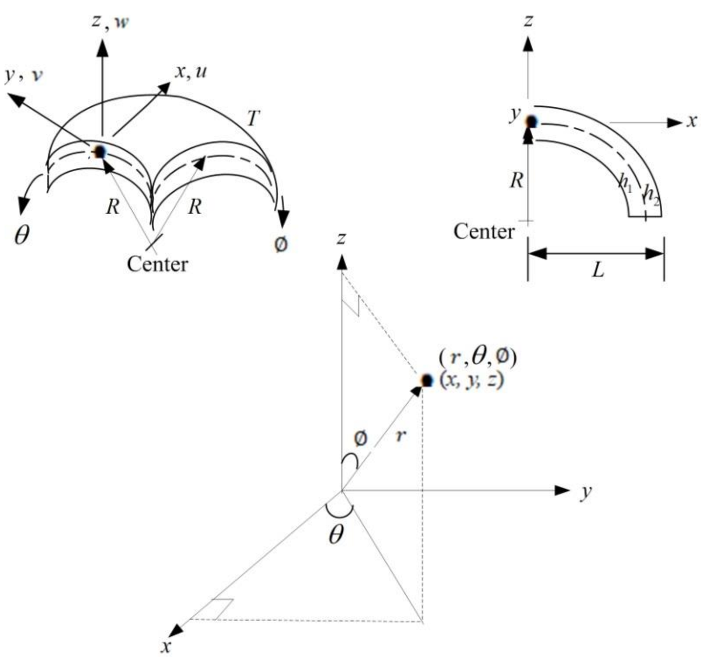


Figure 1. A point $(r, \theta, \emptyset)$ in two-material thick FGM spherical shell for $0° \leqq \emptyset \leqq 90°$ under environment temperature

2.1 Displacement TSDT equations

The time dependent of nonlinear displacements u, v and w of thick FGM spherical shells for a given $\emptyset$ angle are used in c_1 term of TSDT [13] as follows Eq. (1),

$$u = u_0(x,\theta,t) + z\phi_x(x,\theta,t) - c_1 z^3 (\phi_x + \frac{\partial w}{\partial x}),$$

$$v = v_0(x,\theta,t) + z\phi_\theta(x,\theta,t) - c_1 z^3 (\phi_\theta + \frac{\partial w}{R\partial \theta}),$$

$$w = w(x,\theta,t), \quad (1)$$

where u_0 and v_0 are tangential displacements in the in-surface coordinates x and θ axes direction, respectively, w is transverse displacement in the out of surface coordinates z axis direction of the middle-plane of shells, ϕ_x and ϕ_θ are the shear rotations, R is the middle-surface radius of FGM spherical shells, t is time. Coefficient $c_1 = 4/(3h^{*2})$ is used and $h^* = h_1 + h_2$ is the total thickness of spherical shells for two layers of materials.

2.2 Fully homogeneous equation under free vibration

Fully homogeneous equation with TSDT in the elements of thick FGM spherical shells under free vibration in sinusoidal displacement and shear rotations forms for a given $\emptyset$ angle can be used [14][16]. The zero determinant of the coefficient matrix in equation for obtaining non-trivial solution of amplitudes in the five degree of λ_{mn} polynomial equation can be used [14]. When the root of λ_{mn} found and used $\lambda_{mn} = I_0 \omega_{mn}^2$, in which $I_i = \sum_{k=1}^{N^*} \int_{k}^{k+1} \rho^{(k)} z^i dz$, ($i$= 0,1,2,…,6), N^* is total number of layers, $\rho^{(k)}$ is the density of (k) constituent plies. Then the natural frequency ω_{mn} in subscripts mode shape *m* and *n* can be got for a given angle $\emptyset \neq 0$ of thick FGM spherical shells under free vibration. In the coefficients of fully homogeneous equation and λ_{mn} polynomial equation are consisting of stiffness integrals ($A_{i^s j^s}, B_{i^s j^s}, D_{i^s j^s}, E_{i^s j^s}, F_{i^s j^s}, H_{i^s j^s}$, $i^s, j^s = 1,2,6$), ($A_{i^* j^*}, B_{i^* j^*}, D_{i^* j^*}, E_{i^* j^*}, F_{i^* j^*}, H_{i^* j^*}, i^*, j^* = 4,5$) as follows Eq. (2) and c_1 terms [17],

$$(A_{i^s j^s}, B_{i^s j^s}, D_{i^s j^s}, E_{i^s j^s}, F_{i^s j^s}, H_{i^s j^s}) = \int_{-\frac{h^*}{2}}^{\frac{h^*}{2}} \bar{Q}_{i^s j^s} (1, z, z^2, z^3, z^4, z^6) dz$$

$$(A_{i^* j^*}, B_{i^* j^*}, D_{i^* j^*}, E_{i^* j^*}, F_{i^* j^*}, H_{i^* j^*}) = \int_{-\frac{h^*}{2}}^{\frac{h^*}{2}} k_\alpha \bar{Q}_{i^* j^*} (1, z, z^2, z^3, z^4, z^5) dz \quad (2)$$

where k_α is the advanced shear correction coefficient. $\bar{Q}_{i^s j^s}$ and $\bar{Q}_{i^* j^*}$ are the stiffness with z/R terms cannot be neglected are used and assumed in the following simple terms Eq. (3) containing *z*/(R $sin\emptyset$) for thick FGM spherical shells [17],

$$\bar{Q}_{11} = \bar{Q}_{22} = \frac{E_{fgm}}{1 - \nu_{fgm}^2}, \quad \bar{Q}_{12} = \bar{Q}_{21} = \frac{\nu_{fgm} E_{fgm}}{\left(1 + \frac{z}{R sin\emptyset}\right)(1 - \nu_{fgm}^2)},$$

$$\bar{Q}_{44} = \frac{E_{fgm}}{2(1 + \nu_{fgm})}, \quad \bar{Q}_{55} = \bar{Q}_{66} = \frac{E_{fgm}}{2\left(1 + \frac{z}{R sin\emptyset}\right)(1 + \nu_{fgm})},$$

$$\bar{Q}_{16} = \bar{Q}_{26} = \bar{Q}_{45} = 0, \quad (3)$$

in which $\nu_{fgm} = \frac{\nu_1 + \nu_2}{2}$ is the Poisson's ratios of the FGM spherical shells, $E_{fgm} = (E_2 - E_1)(\frac{z + h^*/2}{h^*})^{R_n} + E_1$ is the Young's modulus of the FGM spherical shells, E_1 and E_2 are the Young's modulus, ν_1 and ν_2 are the Poisson's ratios of the FGM constituent material 1 and 2, respectively.

2.3 Advanced k_α expression

For the advanced thick FGM spherical shells study, it is interesting to consider the extra effect of c_1 term of TSDT on the calculation of shear correction coefficient. The advanced k_α expression for a given angle $\emptyset$ in the thick FGM spherical shells can be used in a rational expression form as follows Eq. (4) [15],

$$k_\alpha = \frac{1}{h^*}\frac{FGMZSV}{FGMZIV} \tag{4}$$

in which *FGMZSV* and *FGMZIV* are parameters in functions of E_1, E_2, c_1 and R_n. The values of advanced k_α are in functions of c_1, R_n and T, but not in functions of h^*.

3. Numerical results

In the calculation for choosing and iterating a λ_{mn} into the five degree of λ_{mn} polynomial equation, then the numerical root λ_{mn} can be solved and the numerical ω_{mn} can be got. The composited thick FGM SUS304/Si_3N_4 material is used under T and free vibration. The FGM material 1 is SUS304, the FGM material 2 is Si_3N_4. The geometric values are used basically for $L/R = 1$, $h_1 = h_2$ and $h^* = 1.2$ mm. For calculated values of advanced k_α are varied with c_1, T and R_n from 0.1 to 10 used by Hong [13].

3.1 Non-dimensional frequency

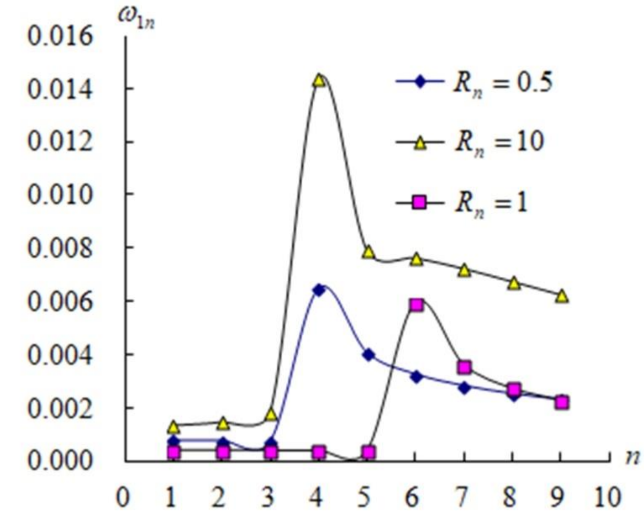
Firstly, for the non-dimensional frequency parameter $f^* = 4\pi\omega_{11}R\sqrt{I_2/A_{11}}$ values under the effects of c_1=0.925925/mm^2 and c_1=0/mm^2 for L/h^*=5, 8 and 10, respectively on $\emptyset = 10^\circ, 45^\circ$ and 90° are shown in Table 1, Table 2 and Table 3, where ω_{11} is the fundamental first natural frequency with subscripts $m = n = 1$. The f^* values under T=1K, 100K, 300K, 600K and 1000K with advanced k_α and c_1=0.925925/mm^2 effects are in the values not greater than 82.795173 on $\emptyset = 10^\circ$, not greater than 8.690546 on $\emptyset = 45^\circ$, not greater than 32.68439 on $\emptyset = 90^\circ$. Another non-dimensional frequency parameter $\Omega = (\omega_{11}L^2/h^*)\sqrt{\rho_1/E_1}$ values under the effects of c_1=0.925925/mm^2 and c_1=0/mm^2 for L/h^*=5, 8 and 10, respectively on $\emptyset = 10^\circ, 45^\circ$ and 90° are shown in Table 4, Table 5 and Table 6, ρ_1 is the density of SUS304, the Ω values under T=1K, 100K, 300K, 600K and 1000K with advanced k_α and c_1=0.925925/mm^2 effects are in the values not greater than 214.34130 on $\emptyset = 10^\circ$, not greater than 14.524525 on $\emptyset = 45^\circ$, not greater than 108.10273 on $\emptyset = 90^\circ$ as shown in Figure 3. It is interesting to compare the present vibration values of frequency with published work as shown in the Table 7 and Table 8. The values of f^* vs. h^* under L/h^*=10 and T=300K with advanced k_α and c_1 effects are shown in Table 7. The compared value f^*=3.011103 at c_1 = 0.925925/mm^2, R_n = 0.5 is smaller than 11.8186 of f^* in three-layer (0°/90°/0°) laminated composite spherical shell, *a*/*h*=10, *R*/*a*=10, TSDT presented by Sayyad and Ghugal in 2019 [16], in which *a* is the arc length in the *x* direction, *h* is the thickness in the *z* direction, *R* is the principal radius in the *x* direction. These different f^* values are mainly caused by different material and advanced k_α. The values of Ω vs. h^* under L/h^*=10 and T=1000K with advanced k_α and c_1 effects are shown in Table 8. The compared value Ω=40.250442 at c_1=0.925925/mm^2, R_n=0.5 is smaller than 69.520 of Ω in composite laminated spherical shell, *h*/*R*=0.02, *n*=*m*=1, SS-SS for four sides simply supported, presented by Li et al. 2019 [18]. These different Ω values are mainly caused by different material and advanced k_α.

3.2 Natural frequency

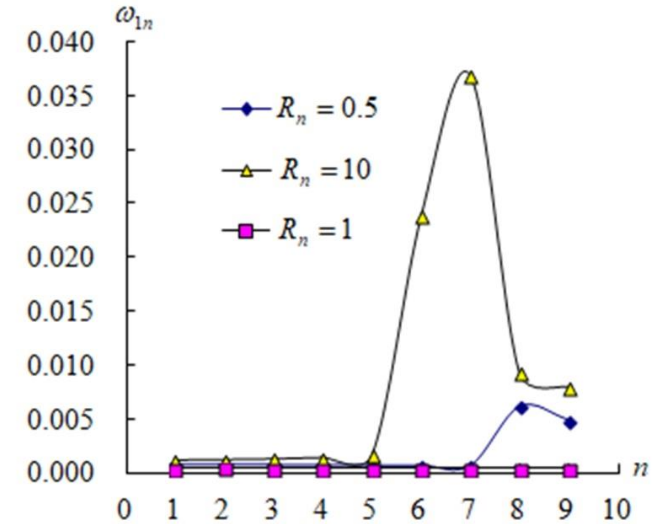
Secondly, the dimensional natural frequency ω_{mn} (1/s) values of free vibration according to two dimensional mode shape of subscripts m and n are calculated. Values of first in $m = n = 1$ fundamental natural frequency ω_{11} vs. R_n, advanced k_α and the effects of c_1=0.925925/mm^2 and c_1=0/mm^2 for L/h^*= 5, 8 and 10, under T=1K, 100K, 300K, 600K and 1000K on $\emptyset = 10^\circ$ are shown in Table 9. The values of ω_{11} are overestimated without the values of c_1, e.g. ω_{11}=0.016143/s with c_1 = 0/mm^2 is greater than ω_{11}=0.001437/s with c_1=0.925925/mm^2 for L/h^*=5, R_n=0.5 under T=1K. For the values of natural frequency ω_{mn} vs. subscripts m, n=1,2,...,9 with R_n=0.5, T=300K under advanced k_α and the effects of c_1=0.925925/mm^2 for L/h^*=5 and 10 on $\emptyset = 10^\circ$ are shown in Table 10. The ω_{mn} values under the effects of advanced k_α and c_1=0.925925/mm^2 are in the values not greater than 0.006482/s for L/h^*=5 and not greater than 0.007896/s for L/h^*=10, respectively on $\emptyset = 10^\circ$.

3.3 Compared ω_{mn}

Finally, the dimensional natural frequency ω_{mn} (1/s) values vs. R_n and T of free vibration according to mode shape of subscripts m=1 and n from 1 to 9 are calculated. Figure 2 shows the values of ω_{1n} vs. R_n for thick L/h^*=5, 10 respectively on $\emptyset = 10^\circ$, with the effects of advanced k_α and c_1=0.925925/mm^2 under T=300K. Generally the values of ω_{1n} are keeping constant firstly then increasing and finally decreasing with values of subscript n from 1 to 9 for L/h^*=5, R_n=1, 0.5 and 10. The greatest value of ω_{14}=0.014334/s is found in Figure 2a for L/h^*=5, R_n=10. The values of ω_{1n} do be affected with R_n for L/h^*=5. The values of ω_{1n} are keeping constant with values of subscript n from 1 to 9 for L/h^*=10, R_n=1; ω_{1n} are almost keeping the lower constant with values of subscript n then increasing and finally decreasing for L/h^*=10, R_n=0.5 and 10, respectively. The greatest value of ω_{17}=0.036801/s is found in Figure 2b for L/h^*=10, R_n=10. The values of ω_{1n} also do be affected with R_n for L/h^*=10.

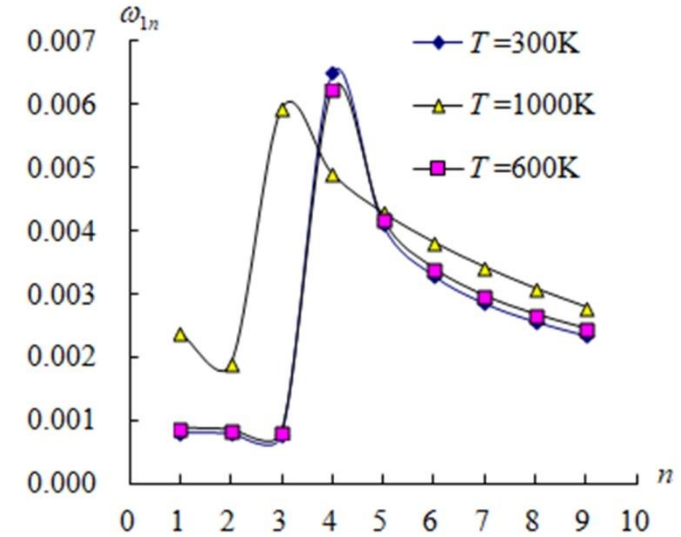


(a) ω_{1n} vs. R_n for L/h^*= 5

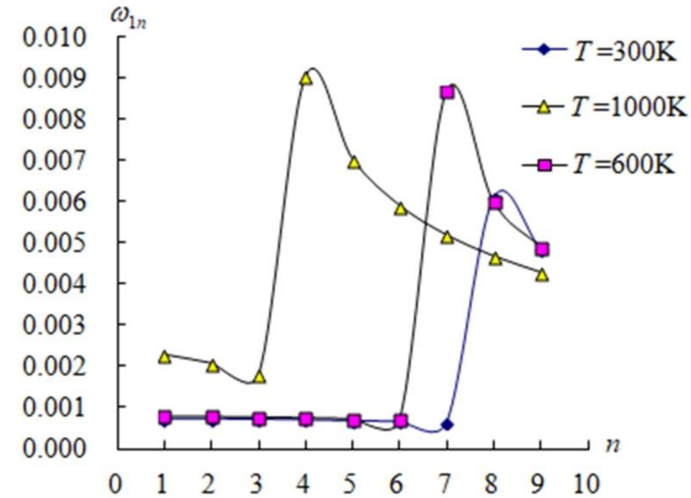


(b) ω_{1n} vs. R_n for L/h^*= 10

Figure 2. ω_{1n} vs. R_n for L/h^*= 5 and 10 on $\emptyset = 10^\circ$ with the effects of advanced k_α and c_1 =0.925925/mm^2 under T=300K.



(a) ω_{1n} vs. T for L/h^*= 5



(b) ω_{1n} vs. T for L/h^*= 10

Figure 3. ω_{1n} vs. T for L/h^*= 5 and 10 on $\emptyset = 10^\circ$ under the effects of advanced k_α, c_1 =0.925925/mm^2 and R_n =0.5

4. Conclusions

The values of natural frequency and frequency parameters respectively on $\emptyset = 10^\circ, 45^\circ$ and 90° are calculated and obtained by using the fully homogeneous equation in the free vibration of thick FGM spherical shells. Three important effect items are nonlinear coefficient term c_1, advanced shear correction coefficient and environment temperature that considered in the frequency calculation and investigation. Important results are contributed and found as follows. Numerical data investigated in the non-dimensional frequency parameters under free vibration with and without the c_1 value. The values of dimensional natural frequency ω_{mn} vs. R_n and T in free vibration according to two dimensional mode shape in subscripts m and n for the thick SUS304/Si_3N_4 FGM spherical shells are calculated under the effects of c_1 value and advanced k_α value.

Conflict of Interest Statement

The author declares that there is no conflict of interest in the study.

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APPENDIX

Table 1 f^* for SUS304/Si_3N_4 on $\emptyset = 10^{\circ}$

L/h^*	R_n	c_1 (1/mm²)	f^* Present solution, advanced k_α T=1K	T=100K	T=300K	T=600K	T=1000K
5	0.5	0.925925	2.820784	2.336545	1.620654	1.827490	5.420166
		0	31.672805	32.410423	33.958835	36.729087	42.579944
	1	0.925925	2.431303	1.865989	0.872658	1.057423	5.053874
		0	34.273555	132.53036	36.110019	39.073677	46.864479
	2	0.925925	1.224839	4.740484	15.659271	16.201717	3.756422
		0	37.479995	37.694744	38.649929	41.847938	52.400417
	10	0.925925	2.943557	2.958956	3.025853	3.291728	4.135724
		0	43.679279	43.131305	43.261871	46.893310	63.765880
8	0.5	0.925925	4.384293	3.617833	2.498185	2.818365	8.539231
		0	79.420143	81.302406	85.231727	92.140853	106.42930
	1	0.925925	3.770107	2.881380	1.341674	1.626429	7.958377
		0	86.240333	87.566551	90.857330	98.279922	117.73574
	2	0.925925	1.886321	11.747584	77.90464	82.795173	5.866292
		0	94.477165	329.58828	97.383865	105.41049	131.93212
	10	0.925925	4.429037	4.457534	4.565095	4.961815	6.188531
		0	109.88312	108.54094	108.90417	117.99297	159.63591
10	0.5	0.925925	5.300385	4.367766	3.011103	3.397310	10.34854
		0	122.11304	125.13778	131.29669	141.78059	161.99548
	1	0.925925	4.551788	3.474352	1.615177	1.957854	9.637208
		0	132.52432	134.76289	139.99891	151.25752	178.71736
	2	0.925925	2.271638	34.964645	2.891051	3.058258	7.084141
		0	144.84071	145.97044	149.92569	162.06086	198.96336
	10	0.925925	5.311273	5.344233	5.471844	5.947144	7.422325
		0	166.86840	871.42504	166.83900	180.36746	235.00473

Table 2 f^* for SUS304/Si_3N_4 on $\emptyset = 45^{\circ}$

L/h^*	R_n	c_1 (1/mm²)	f^* Present solution, advanced k_α T=1K	T=100K	T=300K	T=600K	T=1000K
5	0.5	0.925925	1.591143	1.923344	2.912015	2.954740	1.596829
		0	1.676810	1.839977	2.128164	2.280293	1.946411
	1	0.925925	1.797150	2.311825	8.690546	5.807613	1.726044
		0	1.658154	1.824377	2.116312	2.268444	1.914986
	2	0.925925	3.028405	5.893948	0.607843	0.640845	2.030289
		0	1.702577	1.870286	2.163931	2.320475	1.973235
	10	0.925925	4.431287	4.677474	5.118327	5.449218	5.400196
		0	2.008263	2.163172	2.443043	2.622124	2.432328
8	0.5	0.925925	1.118790	1.311380	1.806936	1.863120	1.156286
		0	0.712809	0.779661	0.894395	0.962980	0.842842
	1	0.925925	1.235278	1.505425	2.763189	2.666691	1.243545
		0	0.698829	0.766372	0.883941	0.951626	0.818396
	2	0.925925	1.766531	0.266276	1.958618	2.068104	1.425871
		0	0.718106	0.786101	0.904854	0.974335	0.841676
	10	0.925925	2.291216	2.416438	2.640617	2.814449	2.796801
		0	0.868164	0.931939	1.044738	1.126561	1.069076
10	0.5	0.925925	1.198938	1.359829	1.752298	1.822630	1.306651
		0	0.487702	0.531283	0.610783	0.657518	0.577413
	1	0.925925	1.292613	1.506000	2.416618	2.389040	1.389970
		0	0.478278	0.522153	0.603453	0.491572	0.560983
	2	0.925925	1.669677	0.318171	1.235356	1.306177	1.534058
		0	0.491173	0.535504	0.617843	0.664538	0.576723
	10	0.925925	1.488171	1.567301	1.708185	1.822322	1.823202
		0	0.594491	0.637972	0.714416	0.770527	0.733014

Table 3 f^* for SUS304/Si_3N_4 on $\emptyset = 90^\circ$

L/h^*	R_n	c_1 (1/mm^2)	f^* Present solution, advanced k_α				
			T=1K	T=100K	T=300K	T=600K	T=1000K
5	0.5	0.925925	0.502130	0.570210	0.745849	0.773258	0.549213
		0	0.590006	0.643852	0.738035	0.792276	0.693156
	1	0.925925	0.542049	0.635356	1.088972	1.061970	0.583895
		0	0.584454	0.639541	0.735365	0.789534	0.682771
	2	0.925925	0.720395	0.147456	1.749742	1.857381	0.643730
		0	0.601933	0.657577	0.754101	0.809998	0.705449
	10	0.925925	0.439585	0.432449	1.912712	2.034272	0.575911
		0	0.714064	0.765051	0.856587	0.920757	0.874146
8	0.5	0.925925	0.384405	0.432271	0.549680	0.572973	0.417923
		0	0.275399	0.300743	0.345240	0.371763	0.325808
	1	0.925925	0.412331	0.475557	0.734396	0.732705	0.444527
		0	0.270439	0.296031	0.341274	0.367443	0.316992
	2	0.925925	0.520187	0.256231	0.966141	1.024323	0.492228
		0	0.278029	0.303717	0.349439	0.376290	0.326585
	10	0.925925	6.133640	0.966218	1.064178	1.131785	1.103627
		0	0.333380	0.359943	0.403599	0.435239	0.415926
10	0.5	0.925925	0.444867	0.489384	0.591412	0.618544	0.501530
		0	0.193089	0.209523	0.242446	0.259834	0.229385
	1	0.925925	0.468606	0.523140	32.68439	0.734637	0.529650
		0	0.184350	0.206358	0.239394	0.257086	0.215634
	2	0.925925	2.502079	0.307402	0.638958	0.678791	0.569559
		0	0.186246	0.211788	0.244852	0.263350	0.215261
	10	0.925925	0.481267	0.507236	0.551137	0.587717	0.588011
		0	0.236454	0.250857	0.283276	0.304129	0.291925

Table 4 Ω for SUS304/Si_3N_4 on $\emptyset = 10^\circ$

L/h^*	R_n	c_1 (1/mm^2)	Ω Present solution, advanced k_α				
			T=1K	T=100K	T=300K	T=600K	T=1000K
5	0.5	0.925925	5.130447	4.151097	2.800106	3.170027	10.540805
		0	57.606563	57.580242	58.672813	63.711536	82.806861
	1	0.925925	4.236908	3.189765	1.458476	1.773653	9.323061
		0	59.726772	226.55050	60.350742	65.539665	86.452575
	2	0.925925	2.036923	7.772025	25.256298	26.214517	6.532387
		0	62.329700	61.800544	62.337131	67.710319	91.123886
	10	0.925925	4.532616	4.533440	4.611013	5.028469	6.503587
		0	67.259231	66.081817	65.925560	71.634582	100.27433
8	0.5	0.925925	12.758659	10.283887	6.906034	7.822132	26.570516
		0	231.11923	231.10652	235.61633	255.72911	331.16351
	1	0.925925	10.511957	7.880797	3.587748	4.364908	23.489767
		0	240.45861	239.50131	242.96005	263.75744	347.50613
	2	0.925925	5.019161	30.816268	201.03955	214.34130	16.322294
		0	251.38674	864.57617	251.30732	272.88812	367.08621
	10	0.925925	10.912035	10.927077	11.130595	12.127532	15.570733
		0	270.72439	266.07431	265.52966	288.39517	401.65393
10	0.5	0.925925	19.280702	15.519516	10.404945	11.786184	40.250442
		0	444.19888	444.63870	453.69909	491.87496	630.07775
	1	0.925925	15.864337	11.878280	5.398899	6.567957	35.55620
		0	461.88674	460.73370	467.96090	507.41922	659.37255
	2	0.925925	7.555527	114.64908	9.325755	9.896577	24.638525
		0	481.74383	478.63714	483.61996	524.43176	691.99127
	10	0.925925	16.357053	16.375883	16.676782	18.169807	23.343793
		0	513.90222	2670.2343	508.48257	551.06146	739.10821

Table 5 Ω for SUS304/Si_3N_4 on $\emptyset = 45^{\circ}$

L/h^*	R_n	c_1 (1/mm²)	Ω Present solution, advanced k_α T=1K	T=100K	T=300K	T=600K	T=1000K
5	0.5	0.925925	2.893975	3.417006	5.031271	5.125394	3.105416
		0	3.049786	3.268897	3.676963	3.955475	3.785260
	1	0.925925	3.131803	3.951888	14.524525	9.741315	3.184095
		0	2.889580	3.118633	3.536997	3.804941	3.532644
	2	0.925925	5.036276	9.663129	0.980370	1.036893	3.530656
		0	2.831408	3.066335	3.490129	3.754549	3.431440
	10	0.925925	6.823486	7.166394	7.799675	8.324268	8.492018
		0	3.092409	3.314214	3.722885	4.005577	3.824931
8	0.5	0.925925	3.255774	3.727669	4.995132	5.170932	3.597878
		0	2.074334	2.216230	2.472485	2.672671	2.622572
	1	0.925925	3.444252	4.117455	7.388997	7.156696	3.670421
		0	1.948502	2.096088	2.363733	2.553913	2.415561
	2	0.925925	4.700423	0.698495	5.054381	5.353939	3.967326
		0	1.910751	2.062102	2.335054	2.522374	2.341868
	10	0.925925	5.644982	5.923592	6.438341	6.878999	7.036926
		0	2.138940	2.284530	2.547276	2.753511	2.689863
10	0.5	0.925925	4.361263	4.831735	6.055112	6.323195	5.082191
		0	1.774068	1.887752	2.110576	2.281108	2.245835
	1	0.925925	4.505143	5.148783	8.077796	8.014444	5.128254
		0	1.666944	1.785161	2.017106	1.978877	2.069731
	2	0.925925	5.553389	1.043285	3.984929	4.226814	5.335431
		0	1.633654	1.755920	1.992995	2.150459	2.005833
	10	0.925925	4.583100	4.802551	5.206111	5.567587	5.734112
		0	1.830847	1.954885	2.177357	2.354128	2.305386

Table 6 Ω for SUS304/Si_3N_4 on $\emptyset = 90^{\circ}$

L/h^*	R_n	c_1 (1/mm²)	Ω Present solution, advanced k_α T=1K	T=100K	T=300K	T=600K	T=1000K
5	0.5	0.925925	0.913275	1.013033	1.288651	1.341320	1.068076
		0	1.073105	1.143866	1.275150	1.374310	1.348008
	1	0.925925	0.944602	1.086094	1.820001	1.781280	1.077132
		0	1.018498	1.093247	1.229017	1.324313	1.259533
	2	0.925925	1.198026	0.241754	2.822099	3.005259	1.119442
		0	1.001023	1.078098	1.216264	1.310585	1.226770
	10	0.925925	0.676893	0.662559	2.914729	3.107570	0.905644
		0	1.099547	1.172142	1.305329	1.406556	1.374629
8	0.5	0.925925	1.118652	1.228756	1.519547	1.590239	1.300403
		0	0.801435	0.854879	0.954389	1.031797	1.013780
	1	0.925925	1.149677	1.300685	1.963837	1.966387	1.312058
		0	0.754050	0.809668	0.912595	0.986122	0.935628
	2	0.925925	1.384125	0.672145	2.493210	2.651783	1.369569
		0	0.739785	0.796712	0.901757	0.974145	0.908687
	10	0.925925	15.111748	2.368561	2.594676	2.766279	2.776796
		0	0.821366	0.882354	0.984053	1.063799	1.046497
10	0.5	0.925925	1.618253	1.738877	2.043641	2.145896	1.950690
		0	0.702381	0.744476	0.837780	0.901436	0.892191
	1	0.925925	1.633234	1.788537	109.2509	2.464468	1.954131
		0	0.642515	0.705509	0.800200	0.862442	0.795578
	2	0.925925	108.10273	1.007971	2.061109	2.196582	1.980920
		0	0.619460	0.694453	0.789826	0.852205	0.748676
	10	0.925925	1.482151	1.554282	1.679727	1.795602	1.849342
		0	0.728204	0.768680	0.863353	0.929181	0.918128

Table 7 Comparison of frequency f^* for SUS304/Si_3N_4 on $\emptyset = 10^\circ$

c_1 (1/mm^2)	h^* (mm)	f^*			
		Present method, L/h^*=10, T=300K, advanced k_α, SUS304/Si_3N_4			Sayyad and Ghugal [16], a/h=10, R/a=10
		$R_n = 0.5$	$R_n = 1$	$R_n = 2$	
0.925925	1.2	3.011103	1.615177	2.891051	11.8186
0.333333	2	23.323301	15.971736	9.100513	-
0.000033	200	15716.966	11534.686	9898.7031	-
0.000014	300	28897.183	21148.072	18228.677	-
0.000003	600	82492.976	60802.226	49333.960	-
0.000001	900	154100.14	110533.68	93945.585	-

Table 8 Comparison of frequency Ω for SUS304/Si_3N_4 on $\emptyset = 10^\circ$

c_1 (1/mm^2)	h^* (mm)	Ω			
		Present method, L/h^*=10, T=1000K, advanced k_α, SUS304/Si_3N_4			Li et al. [18], h/R=0.02,
		$R_n = 0.5$	$R_n = 1$	$R_n = 2$	
0.925925	1.2	40.250442	35.556209	24.638525	69.520
0.333333	2	2474.4003	1039.7077	70.613998	-
0.000033	200	-	-	-	-
0.000014	300	-	-	-	-
0.000003	600	-	-	-	-
0.000001	900	-	-	-	-

Table 9 ω_{11} for advanced k_α on $\emptyset = 10^\circ$

L/h^*	R_n	c_1 (1/mm^2)	ω_{11}				
			T=1K	T=100K	T=300K	T=600K	T=1000K
5	0.5	0.925925	0.001437	0.001177	0.000797	0.000865	0.002389
		0	0.016143	0.016328	0.016713	0.017395	0.018770
	1	0.925925	0.001187	0.000904	0.000415	0.000484	0.002113
		0	0.016737	0.064243	0.017191	0.017894	0.019597
	2	0.925925	0.000570	0.002203	0.007194	0.007157	0.001480
		0	0.017467	0.017524	0.017757	0.018486	0.020655
	10	0.925925	0.001270	0.001285	0.001313	0.001372	0.001474
		0	0.018848	0.018738	0.018779	0.019558	0.022730
8	0.5	0.925925	0.001396	0.001139	0.000768	0.000834	0.002352
		0	0.025299	0.025599	0.026217	0.027274	0.029323
	1	0.925925	0.001150	0.000872	0.000399	0.000465	0.002079
		0	0.026322	0.026529	0.027034	0.028130	0.030770
	2	0.925925	0.000549	0.003413	0.022370	0.022859	0.001445
		0	0.027518	0.095769	0.027963	0.029104	0.032504
	10	0.925925	0.001194	0.001210	0.001238	0.001293	0.001378
		0	0.029635	0.029473	0.029546	0.030757	0.035565
10	0.5	0.925925	0.001350	0.001100	0.000740	0.000804	0.002280
		0	0.031120	0.031521	0.032309	0.033574	0.035706
	1	0.925925	0.001111	0.000842	0.000384	0.000448	0.002014
		0	0.032359	0.032662	0.033325	0.034635	0.037366
	2	0.925925	0.000529	0.008127	0.000664	0.000675	0.001396
		0	0.033750	0.033931	0.034440	0.035796	0.039215
	10	0.925925	0.001145	0.001160	0.001187	0.001240	0.001322
		0	0.036003	0.189300	0.036211	0.037613	0.041885

Table 10 ω_{mn} vs. m and n for advanced k_{α}, c_1, $R_n = 0.5$ and T=300K on $\emptyset = 10^{\circ}$

c_1 (1/mm^2)	L/h^*	ω_{1n}								
		$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$
0.925925	5	0.000797	0.000783	0.000752	0.006482	0.004085	0.003281	0.002841	0.002548	0.002328
	10	0.000740	0.000738	0.000729	0.000714	0.000694	0.000670	0.000643	0.006078	0.004855
	L/h^*	ω_{2n}								
		$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$
	5	0.000572	0.000556	0.000529	0.005008	0.003541	0.002924	0.002574	0.002343	0.002176
	10	0.000457	0.000453	0.000444	0.000433	0.000419	0.000403	0.006604	0.004884	0.004041
	L/h^*	ω_{3n}								
		$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$
	5	0.000524	0.000508	0.007423	0.003973	0.003031	0.002572	0.002299	0.002117	0.001984
	10	0.000366	0.000362	0.000354	0.000344	0.000333	0.007896	0.005026	0.003960	0.003366
	L/h^*	ω_{4n}								
		$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$
	5	0.000517	0.000500	0.004858	0.003212	0.002576	0.002237	0.002028	0.001886	0.001782
	10	0.000326	0.000322	0.000315	0.000306	0.000296	0.005535	0.003978	0.003264	0.002833
	L/h^*	ω_{5n}								
		$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$
	5	0.000528	0.007756	0.003626	0.002636	0.002188	0.001937	0.001778	0.001668	0.001588
	10	0.000306	0.000302	0.000296	0.000287	0.007862	0.004218	0.003251	0.002744	0.002420
	L/h^*	ω_{6n}								
		$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$
	5	0.000552	0.004778	0.002844	0.002185	0.001860	0.001673	0.001554	0.001471	0.001411
	10	0.000295	0.000291	0.000285	0.000277	0.005087	0.003386	0.002729	0.002350	0.002098
	L/h^*	ω_{7n}								
		$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$
	5	0.000593	0.003436	0.002277	0.001814	0.001578	0.001442	0.001355	0.001295	0.001252
	10	0.000288	0.000285	0.000279	0.000271	0.003827	0.002821	0.002341	0.002046	0.001844
	L/h^*	ω_{8n}								
		$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$
	5	0.005514	0.002590	0.001818	0.001491	0.001326	0.001233	0.001176	0.001137	0.001109
	10	0.000285	0.000281	0.000275	0.006030	0.003103	0.002414	0.002043	0.001805	0.001639
	L/h^*	ω_{9n}								
		$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$
	5	0.003625	0.001917	0.001388	0.001176	0.001084	0.001037	0.001010	0.000992	0.000978
	10	0.000283	0.000279	0.000273	0.003911	0.002623	0.002107	0.001808	0.001611	0.001471